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MATHEW as applied in the AirQUIS System

Model description

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Summary

The diagnostic wind field model MATHEW (Sherman, 1978; Foster et al., 1995) is presently used (in addition to the MM5 model) as wind field generator for the Air Quality Information System, AirQUIS.

MATHEW is a diagnostic wind field model able to generate a 3-dimensional wind field in a Cartesian grid, based on an arbitrary number of wind observations within the model domain. The model treats variable topography within the model domain, and computes a wind field that minimizes the variances between the observations and the calculated values. In addition the computed wind field is mass conserving, e.g. a condition that is approximated in the model by requiring the flow field to be free of divergence ($\nabla \cdot \vec{V} = 0$).

Since MATHEW applies a Cartesian grid, resulting in a box-shaped representation of the model topography, and the dispersion model (AirQUIS/ EPISODE; hereafter just denoted EPISODE) applies a stretched terrain following vertical σ-coordinate, the MATHEW calculated wind field can not be transferred directly onto the EPISODE model grid. Furthermore, the box-formed topography in MATHEW often leads to artificial stagnant conditions close to the ground with calculated wind speeds that are much too low (Slørdal, 2001). For applications in dispersion modelling this is obviously a serious flaw since it may lead to an overestimation of the calculated ground level concentrations in the stagnant areas. In an attempt to reduce the problems associated with this unfavourable feature, a method has been developed in which the MATHEW calculated wind is post-processed before it is exported to the dispersion model.

In detail the applied method can be described as follows: Linear interpolation is applied on each of the grid columns in MATHEW in order to calculate the horizontal wind components, *U* and *V*, at the heights above ground corresponding to the midpoints of the EPISODE layers. These values will here be denoted: $U_{i,j,k}^{Ep}$ and $V_{i,j,k}^{Ep}$. In these expressions *i* and *j* indicate horizontal grid position, while *k* denotes the layer above ground in the EPISODE model. *U*- and *V*-values with equal *k*-index are therefore valid for the same value of the terrain following (σ-) coordinate in the EPISODE model, even though their height relative to mean sea level might be very different.

Direct application of $U_{i,j,k}^{Ep}$ and $V_{i,j,k}^{Ep}$ in EPISODE would still have lead to terrain induced stagnation locally, and therefore the following smoothing algorithm has been introduced:

$$U_{i,j,k}^{NEW} = \left(\frac{1}{4} U_{i-1,j,k}^{Ep} + \frac{1}{2} U_{i,j,k}^{Ep} + \frac{1}{4} U_{i+1,j,k}^{Ep} \right) \qquad \text{(A)}$$

$$V_{i,j,k}^{NEW} = \left(\frac{1}{4} V_{i,j-1,k}^{Ep} + \frac{1}{2} V_{i,j,k}^{Ep} + \frac{1}{4} V_{i,j+1,k}^{Ep} \right) \qquad \text{(B)}$$

$U_{i,j,k}^{NEW}$ and $V_{i,j,k}^{NEW}$ are then transferred to EPISODE without further interpolation. The smoothing method in eqs. (A) and (B) effectively reduces the clear signs of topographical induced underestimation and directional distortion of the calculated wind field.

MATHEW as applied in the AirQUIS System
Model description

1 Introduction

The diagnostic wind field model MATHEW (Sherman, 1978; Foster et al., 1995) is presently used (in addition to the MM5 model) as wind field generator for the Air Quality Information System, AirQUIS.

Since MATHEW applies a Cartesian grid, resulting in a box-shaped representation of the model topography, and the dispersion model (AirQUIS/ EPISODE; hereafter just denoted EPISODE) applies a stretched terrain following vertical σ-coordinate, the MATHEW calculated wind field can not be transferred directly onto the EPISODE model grid. Furthermore, the box-formed topography in MATHEW often leads to artificial stagnant conditions close to the ground with calculated wind speeds that are much too low (Slørdal, 2001). For application in dispersion modelling this is obviously a serious flaw since it might lead to an overestimation of the calculated ground level concentrations in the stagnant areas. In an attempt to reduce the problems associated with this unfavourable feature, a method has been developed in which the MATHEW calculated wind is post-processed before it is exported to the dispersion model.

In this technical report a brief description of the MATHEW model is presented in Section 2, and a detailed description of the procedure for transferring the MATHEW calculated wind into the grid system of the dispersion model (EPISODE) is presented in Section 3.

2 Brief description of MATHEW

MATHEW is a diagnostic wind field model able to generate a 3-dimensional wind field in a Cartesian grid, based on a arbitrary number of wind observations within the model domain. The model treats variable topography within the model domain, and computes a wind field that minimizes the variances between the observations and the calculated values. In addition the computed wind field is mass conserving, e.g. a condition that is approximated in the model by requiring the flow field to be free of divergence ($\nabla \cdot \vec{V} = 0$).

The applied method can be briefly described as follows: At the outset the ground is considered flat. All of the available surface wind observations (at least one must be present) are then normalized to a specified *reference height* above the ground, Z_{ref}, by use of an idealized profile function The reference height is normally chosen equal to the most commonly occurring measurement height, typically 10m. Based on the observed wind vectors a gridded 2-dimensional wind field is then calculated by an interpolation procedure. The applied interpolation procedure calculates a wind vector in each grid point based on a weighted sum of the nearest measurement values, where the weight decrease with the square of the distance to the observation value.

A similar procedure is applied for the calculation of a horizontal wind field near the upper boundary of the model domain, Z_{top}. This wind field is either computed from existing upper air measurements or from available model calculated values from NWP models, or from both. If upper air data are not available, one upper air value is computed by applying an idealised vertical profile function to one of the surface observations. The surface station that is considered to be the least influenced by local topography is chosen. By applying the same interpolation procedure as for the surface wind, a 2-dimensional wind field is then constructed at $z = Z_{top}$. Then the wind values in the grid points between Z_{ref} and Z_{top} are computed using stability dependent, vertical profile functions. In this way all of the grid points within the model domain are assigned an initial wind vector.

After the initial wind has been assigned, the topography is introduced. This is accomplished simply by elevating each vertical grid column by an even number of vertical grid distances, $n \cdot \Delta Z$, corresponding to the height of the topography ($\pm\Delta Z/2$) at the position of each grid column. The grid points that are pushed out of the top model boundary are just removed from the model domain. This procedure leads to a box-shaped topography, where each box is rectangular with a volume of $\Delta X \cdot \Delta Y \cdot (n \cdot \Delta Z)$. Here ΔX and ΔY are the constant grid distances in the x- and y-direction, while $n \cdot \Delta Z$ is an integer number of the equally spaced vertical levels in the model.

The resulting wind field, (u^0,v^0,w^0), is then modified as slightly as possible, in order to minimize the variance between the observations and the calculated values, and at the same time constrain the flow so that it becomes volume conserving (i.e. divergence-free; $\nabla \cdot \vec{V} = 0$)*. This modification is achieved by solving the following variational problem where the functional to be minimized is given by (Sherman, 1978; Foster et al., 1995):

$$J(u,v,w,\lambda) = \int_{\Omega} \frac{1}{2\sigma_H^2}\left[\left(u-u^0\right)^2 + \left(v-v^0\right)^2\right] + \frac{1}{2\sigma_V^2}\left(w-w^0\right)^2 + \lambda\left(\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} + \frac{\partial w}{\partial z}\right) dxdydz \quad (1)$$

where (u,v,w) are the calculated components of the wind field and (u^0,v^0,w^0) are the initially interpolated wind field, $\lambda(x,y,z)$ is the Lagrange's multiplier for the constraint of mass consistency, i.e. $\nabla \cdot \vec{V} = 0$, Ω is the model domain, and σ_H and σ_V are stability parameters that control the degree of horizontal or vertical adjustment of the flow field. Spatially constant values of σ_H and σ_V are used for each hour, and their values depend on estimated values of the Monin-Obukhov length. This stability parameter is calculated using the meteorological pre-processor MEPDIM (Bøhler, 1996) from measured input of wind speed and vertical temperature difference at one of the measurement stations (the main station).

The goal is to find the 3-dimensional wind field, u, v and w, that gives the minimum value of J.

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* In the MATHEW formulation the continuity equation is simplified to: $\nabla \cdot \vec{V} = 0$, and this restriction on the flow therefore corresponds to the requirement of mass conservation.

The Euler-Lagrange-equations associated with the functional *J* are given by:

$$u-u^0=\sigma_H^2\frac{\partial\lambda}{\partial x},\qquad v-v^0=\sigma_H^2\frac{\partial\lambda}{\partial y}\qquad\text{and}\qquad w-w^0=\sigma_V^2\frac{\partial\lambda}{\partial z}\tag{2}$$

with boundary conditions:

$$\lambda\, n_x\delta\, u\big|_s=0,\qquad \lambda\, n_y\delta\, v\big|_s=0\qquad\text{and}\qquad \lambda\, n_z\delta\, w\big|_s=0\tag{3}$$

where n_x is the component of the boundary normal vector in the direction of the x-axis and $\delta u\,\big|_s$ is the variation of the wind speed in this direction at the boundary, and similarly for the two other directions

Taking the derivative with respect to x of the u-equation in (2), the y-derivative of the v-equation and the z-derivative of the w-equation, then summing the equations and utilizing that $\nabla\cdot\vec{v}=0$, we end up with the following Poisson equation for $\lambda(x,y,z)$.

$$\sigma_H^2\left(\frac{\partial^2\lambda}{\partial x^2}+\frac{\partial^2\lambda}{\partial y^2}\right)+\sigma_V^2\frac{\partial^2\lambda}{\partial z^2}=-\left(\frac{\partial u^0}{\partial x}+\frac{\partial v^0}{\partial y}+\frac{\partial w^0}{\partial z}\right)\tag{4}$$

This equation is then solved for $\lambda(x,y,z)$ applying the given boundary conditions in (3), and finally the wanted wind field is found by insertion in (2). The equations are expressed in finite difference form, and the resulting set of linear equations for the grid values of λ are then solved by an iterative solution procedure. The iteration procedure is terminated when the absolute value of the relative change of all the λ values are less than a predefined limit value. For a more detailed description of the method, see Foster et al., 1995; pp. 1-14.

One of the main problems with applying the MATHEW calculated wind field in dispersion calculations is linked to the box-shaped topography. In areas where the real topography is gently sloping over considerable distances, the MATHEW topography will typically be specified as horizontal with sudden steps in elevation. When investigating the model output from MATHEW, the calculated wind speed close to ground is often too low near these artificial discontinuities in the topography. In addition the wind direction becomes distorted as well, showing extensive occurrences of low wind directed either in the x- or y-direction, i.e. along the vertical sides of the topography boxes (Slørdal, 2001). Consequently, if the MATHEW calculated wind is applied directly as input to a dispersion model, overestimations of ground level concentrations are to be expected in areas with abrupt changes in the model topography. In order to reduce these problems, the MATHEW calculated wind is slightly modified by a post-processing procedure before it is exported to the dispersion model. This procedure is described in the next section.

3 Description of the procedure for extracting the wind fields from MATHEW to the AirQUIS/EPISODE System

The problems associated with the abrupt changes in the MATHEW topography were in earlier AirQUIS/EPISODE applications reduced by applying finer resolution in MATHEW than in the dispersion model. Typically the dispersion calculations were performed in a horizontal grid twice the size of the MATHEW grid. The wind field was then interpolated both horizontally and vertically to the required grid points in the dispersion model.

Motivated by the desire to minimize the computational costs and, at the same time, reduce the amount of superfluous interpolation, a new procedure has been designed for transferring gridded wind fields between MATHEW and AirQUIS/-EPISODE. This procedure permits identical horizontal resolutions in both models. It should be noted that only the horizontal components of the MATHEW wind field are transferred to the EPISODE grid. The vertical wind component is then re-calculated in the dispersion model by requiring $\nabla \cdot \vec{v} = 0$ to be fulfilled for each grid cell separately.

In detail the applied method can be described as follows: Linear interpolation is applied on each of the grid columns in MATHEW in order to calculate the horizontal wind components, *U* and *V*, at the heights above ground corresponding to the midpoints of the EPISODE layers. These values will here be denoted: $U_{i,j,k}^{Ep}$ and $V_{i,j,k}^{Ep}$. In these expressions *i* and *j* indicate horizontal grid position, while *k* denotes the layer above ground in the EPISODE model. *U*- and *V*-values with equal *k*-index are therefore valid for the same value of the terrain following (σ-) coordinate in the EPISODE model, even though their height relative to mean sea level may be very different.

Direct application of $U_{i,j,k}^{Ep}$ and $V_{i,j,k}^{Ep}$ in EPISODE would still have lead to terrain induced stagnation locally, and therefore the following smoothing algorithm has been introduced:

$$U_{i,j,k}^{NEW} = \left(\frac{1}{4} U_{i-1,j,k}^{Ep} + \frac{1}{2} U_{i,j,k}^{Ep} + \frac{1}{4} U_{i+1,j,k}^{Ep} \right) \tag{5}$$

$$V_{i,j,k}^{NEW} = \left(\frac{1}{4} V_{i,j-1,k}^{Ep} + \frac{1}{2} V_{i,j,k}^{Ep} + \frac{1}{4} V_{i,j+1,k}^{Ep} \right) \tag{6}$$

$U_{i,j,k}^{NEW}$ and $V_{i,j,k}^{NEW}$ are then transferred to EPISODE without further interpolation. The boundary values of $U_{i,j,k}^{NEW}$ and $V_{i,j,k}^{NEW}$ are put equal to the neighbouring grid value in the model interior. The smoothing method in eqs. (5) and (6) effectively reduces the clear signs of topographically induced underestimation and directional distortion of the calculated wind field. This is clearly demonstrated in Figure 1 and 2 below. Figure 1 shows the computed unsmoothed wind field for the lowermost EPISODE layer, while the corresponding smoothed wind field is presented in Figure 2. Furthermore, tests have revealed that the quality of the transferred wind field when applying the simple smoothing in eqs. (5) and (6), is equally good, and in some cases even better, than the previously cumbersome method which required twice as fine a grid resolution in MATHEW as in EPISODE (Slørdal, 2001).

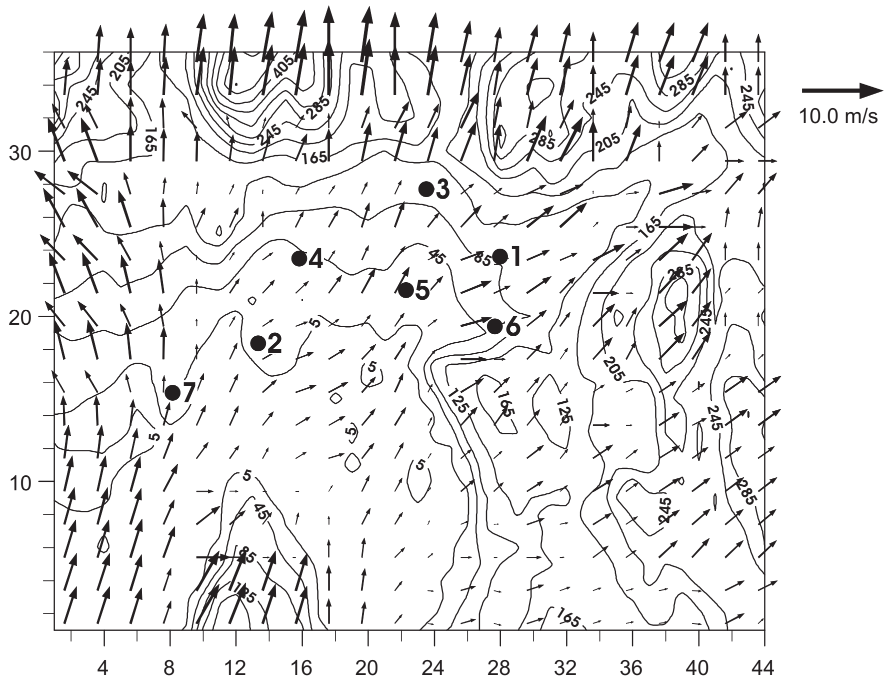

Figure 1: Horizontal wind field in the lowermost EPISODE layer above ground when the MATHEW calculated wind field is transferred without any smoothing. This calculation has been made in a 22 km x 18 km area covering the city of Oslo. The contour lines shows the topography of the model area. The numbered bullets indicate the positions of the applied wind measurements.

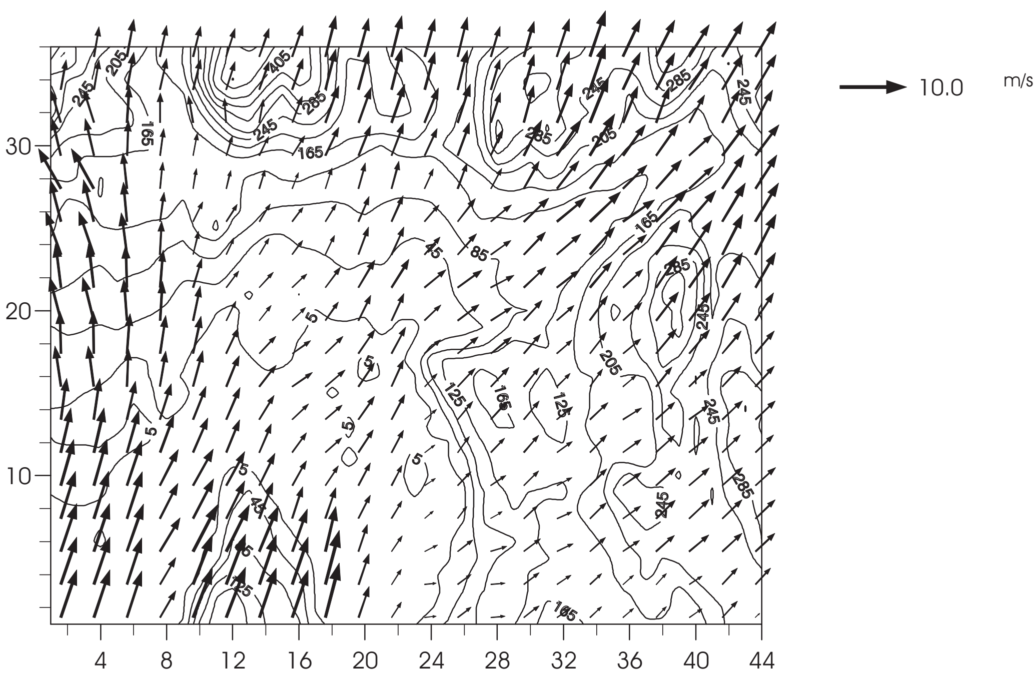

Figure 2: The same situation as in Figure 1, but with the smoothing procedure in eqs. (5) and (6) included.

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ABSTRACT

The diagnostic wind field model MATHEW (Sherman, 1978; Foster et al., 1995) is presently used as wind field generator for the Air Quality Information System, AirQUIS. In this technical report a brief mathematical description of MATHEW is presented. In addition a detailed description of the procedure for transferring the MATHEW calculated wind into the grid system of the dispersion model (AirQUIS/EPISODE) is presented.

NORWEGIAN TITLE

Mathew anvendt i AirQUIS.

KEYWORDS		
Wind field model MATHEW	Model description	AirQUIS

ABSTRACT (in Norwegian)

Den diagnostiske vindfelt-modellen MATHEW (Sherman, 1978; Foster et al., 1995) bukes i dag som vindfelt generator i det integrerte luftkvalitetsystemet AirQUIS. I denne tekniske rapporten presenteres en kortfattet matematisk beskrivelse av MATHEW. I tillegg gis en mer detaljert beskrivelse av prosedyren som benyttes for å overføre den MATHEW-beregnede vinden til gittersystemet som anvendes i spredningsmodellen (AirQUIS/EPISODE).

** Classification*	*A*	*Unclassified (can be ordered from NILU)*
	B	*Restricted distribution*
	C	*Classified (not to be distributed)*